

Two-dimensional Hawking radiation from the AdS/CFT correspondence

Jorge S. Díaz*

Physics Department, Indiana University, Bloomington, Indiana 47405, USA

The AdS/CFT correspondence has been tested through the reproduction of standard results. Following this approach, we use the correspondence to obtain the Hawking temperature of a black hole in 1+1 dimensions. Using an auxiliary Liouville field, the holographic energy-momentum tensor is found and compared with a radiation energy-momentum tensor, verifying that the correspondence gives the correct temperature. The information about the radiated field in the CFT sector is contained in the central charge, whereas in the radiation tensor this information is in the statistical distribution. This result allows to determine the radiation of scalar and Dirac fields easily and without the necessity of solving the corresponding Klein-Gordon or Dirac equation in a curved spacetime. In both cases, the correct temperature was obtained.

I. INTRODUCTION

Since the establishment of the two governing theories of modern physics, Quantum Mechanics and the General Theory of Relativity, a search has been underway for a unification of these theories. The goal of which would be understanding the short-range behavior of gravity. The result, a quantum theory of gravity, would be key to understanding such phenomena as the behavior of particles during the early universe, or the mechanism of Hawking radiation [1].

Over the last few decades, superstring theory has become the most promising framework to describe a unified picture of the known interactions. One of the breakthroughs of last ten years in this field was the AdS/CFT correspondence [2]. This duality establishes a one-to-one correspondence between the states of a five-dimensional superstring theory in a Anti-de Sitter (AdS_5) spacetime and a four-dimensional conformal field theory (CFT_4) laying at the boundary of the AdS manifold. This can be extended to a different dimensions as an $\text{AdS}_{d+1}/\text{CFT}_d$ correspondence. In the low energy limit the superstring theory reduces to a gravitational theory, thus AdS/CFT becomes a dictionary between a $(d+1)$ -dimensional gravitational theory and a d -dimensional quantum theory. In this sense, AdS/CFT offers the possibility to understand quantum aspects of gravity. This duality has been verified only on a case-by-case basis and until now, though mathematically quite appealing, remains as a conjecture. Several authors have used the correspondence to reproduce standard results in order to try to understand its origins as well as test the consistency of the conjecture. In this vein, we set out to determine the Hawking temperature of a two-dimensional black hole is determined through the holographic energy-momentum tensor calculated using $\text{AdS}_3/\text{CFT}_2$.

In the next section, the procedure to identify the temperature from the energy-momentum tensor and how the AdS/CFT correspondence is used to determine this ten-

sor is discussed. Section III contains the explicit calculation of the holographic tensor; whereas the calculation of the radiation tensor is presented in section IV. In the final section the results are summarized and a possible extension to a four-dimensional black hole is outlined.

II. THE ENERGY-MOMENTUM TENSOR

A. The AdS/CFT correspondence and the energy-momentum tensor

In quantum field theory, the partition function is written as a path-integral over all fields Φ as follows

$$\mathcal{Z} = \int \mathcal{D}\Phi e^{I[\Phi]}, \quad (1)$$

which is a useful function to determine expectation values of physical quantities by coupling the desired quantity to its corresponding current in the action $I[\Phi]$. The object of our interest is the energy-momentum tensor, which can be obtained by coupling the action to the metric

$$\langle T_{ij} \rangle = \frac{\delta \mathcal{Z}}{\delta g^{ij}} = \int \mathcal{D}\Phi T_{ij} e^{I[\Phi, g]}. \quad (2)$$

One aspect of the AdS/CFT correspondence establishes that the exponential of the $(d+1)$ -dimensional gravity action as a function of the induced metric of its boundary is exactly the partition function of a d -dimensional conformal field theory lying on that boundary spacetime

$$e^{I_{\text{AdS}}[g_{(0)}]} = \int \mathcal{D}\Phi e^{I_{\text{CFT}}[\Phi, g_{(0)}]}. \quad (3)$$

In order to understand the way to use this correspondence, let's look at the left side in more detail. In this work we shall study a two-dimensional quantum theory and its correspondence to a three-dimensional dual gravity theory. In the low energy limit, the superstring theory in AdS_3 reduces to Einstein's equations with negative

*Electronic address: jsdiazpo@indiana.edu

cosmological constant, hence the three-dimensional metric satisfies

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \quad (4)$$

where the cosmological constant is related with AdS_3 radius by $\Lambda = -l^{-2}$, and Greek indices are used for the bulk (2+1 dimensional) solution, whereas Latin indices are used for boundary induced (1+1 dimensional) spacetime. The solution of the differential equation (4) is the bulk-metric $g_{\mu\nu}(z, x)$, which can be written as a function of its boundary condition determined by the two-dimensional metric $g_{(0)ij}(x)$. Therefore, the three-dimensional action is reduced to a two-dimensional action

$$I_3[g(g_{(0)})] \rightarrow I_2[g_{(0)}], \quad (5)$$

thus, using (2) and (3), the expectation value of the energy-momentum tensor describing the quantum theory can be obtained varying the gravitational action with respect to the boundary metric

$$\langle T_{ij} \rangle_{\text{CFT}} = \frac{\delta I_2}{\delta g_{(0)}^{ij}}. \quad (6)$$

B. 1+1 Black hole temperature and the energy-momentum tensor

In two dimensions, the conformal anomaly and the energy-momentum conservation

$$T^\mu{}_\mu = \frac{c}{24\pi} R, \quad \nabla_\mu T^\mu{}_\nu = 0 \quad (7)$$

completely determine the regularized energy-momentum tensor. By comparing this tensor with that representing a radiation flux, the temperature can be identified. This procedure has been followed by several authors [3, 4, 5] for massless scalar fields and the temperature found agrees with the one predicted by Hawking [1].

In this work the procedure for finding the temperature by comparing the energy-momentum tensor of a quantum field in a black hole background with the one representing a thermal flux is the same as followed by authors above; nonetheless, the way to calculate the expectation value of the energy-momentum tensor will be by invoking AdS/CFT.

III. HOLOGRAPHIC ENERGY-MOMENTUM TENSOR

The three-dimensional AdS spacetime will be described by a element of line written in Fefferman-Graham coordinates [6], as follows

$$ds^2 = \frac{l^2}{z^2} \left(dz^2 + g_{ij}(x, z) dx^i dx^j \right). \quad (8)$$

These coordinates allow for the expansion of the two-dimensional metric $g_{ij}(x, z)$ as power series

$$g_{ij}(x, z) = g_{(0)ij}(x) + z^2 g_{(1)ij}(x) + z^4 g_{(2)ij}(x). \quad (9)$$

Generically this expansion is infinite; nevertheless, in three dimensions the Weyl tensor always vanishes hence the metric is conformally flat and the FG expansion becomes finite. Additionally, as the boundary of the three-dimensional manifold is defined at $z = 0$, this expansion allows for the identification the induced two-dimensional metric $g_{(0)ij}$.

It was shown in [7] that the AdS/CFT correspondence gives, after the regularization of the gravitational action, the expectation value of the energy-momentum tensor in terms of the coefficients of the FG expansion as

$$\langle T_{ij} \rangle = \frac{l}{8\pi G_3} \left(g_{(1)ij} - g_{(0)ij} \text{Tr} g_{(1)} \right), \quad (10)$$

where G_3 is the Newton constant in three dimensions. The energy-momentum tensor of the conformal theory is completely determined by the metric induced at boundary $g_{(0)}$. In general, all terms $g_{(k)}$ can be written in terms of $g_{(0)}$ because this first coefficient of the FG expansion is the boundary condition for equation (4). For the particular case of $k = d/2$ this dependence is non-local and when $d = 2$ Einstein's equations only fix its trace [8]:

$$\text{Tr} g_{(1)} = \frac{1}{2} R_{(0)}, \quad (11)$$

where $R_{(0)}$ is the curvature of $g_{(0)}$. This property was used by Skenderis and Solodukhin [8] to introduce a Liouville field as an auxiliary field, in which case the energy-momentum tensor reads

$$\langle T_{ij} \rangle = \frac{l}{16\pi G_3} \left[\frac{1}{2} \nabla_i \phi \nabla_j \phi - \nabla_i \nabla_j \phi + g_{(0)ij} \left(\square \phi - \frac{1}{4} \nabla^k \phi \nabla_k \phi \right) \right], \quad (12)$$

where the covariant derivative is taken using $g_{(0)}$ and the auxiliary field ϕ satisfies the Liouville field equation without potential

$$\square \phi = R_{(0)}. \quad (13)$$

The three-dimensional Newton constant G_3 and the AdS radius l can be related to the central charge of the CFT through the Brown-Henneaux [9] central charge

$c = 3l/2G_3$. Thus the energy-momentum tensor only depends on the boundary metric and the CFT is characterized by the central charge. With this the correct conformal anomaly (7) can be obtained [10, 11]. This energy-momentum tensor then becomes the same that is found in [12] for a scalar field when $c = 1$ is chosen.

In order to describe the energy-momentum tensor in a black hole background, the boundary metric is written as

$$ds_{1+1}^2 = g_{(0)ij} dx^i dx^j = -f(r) dt^2 + \frac{dr^2}{f(r)}. \quad (14)$$

This metric features an event horizon $r = r_+$ with $f(r_+) = 0$ and $f'(r_+) \neq 0$. The two integration constants obtained when (13) is solved are fixed by imposing regularity of the energy-momentum tensor in the future horizon \mathcal{H}^+ , in order to get a particle flow at infinity which is only described by the Unruh vacuum state [4]. After fixing these constants it is possible to show that, if the metric (14) is asymptotically flat at infinity, all the components of the tensor (12) converge to the same value given by

$$\langle T^{ij} \rangle = \frac{c f'^2(r_+)}{192\pi}, \quad i, j = 0, 1 \quad (15)$$

which is the tensor needed to identify the temperature of a flux of particles at infinity. In the works mentioned above [3, 4, 12] the energy-momentum tensor was calculated for a massless scalar field, whose central charge is $c = 1$. An important feature of result (15) is the presence of the central charge, because it distinguishes between scalar and Dirac fields when it is treated as the parameter of the theory [13].

IV. RADIATION ENERGY-MOMENTUM TENSOR AND HAWKING TEMPERATURE

In this section we show the calculation of the energy-momentum tensor of a radiated field with statistical distribution $\langle n_\omega \rangle$. It is possible to show that for massless fields the magnitude of energy density, flux, and radiation pressure are exactly the same ($\rho = F = P$); therefore, all the energy-momentum tensor components are equal $T^{ij} = \rho$ [14]. In two dimensions, the energy density for a given type of field is

$$\rho = \frac{1}{2\pi} \int_0^\infty \omega \langle n_\omega \rangle d\omega. \quad (16)$$

If we have Υ fields, the components of the total radiation energy-momentum tensor will be simply given by

$$T^{ij} = \frac{\Upsilon}{2\pi} \int_0^\infty \omega \langle n_\omega \rangle d\omega, \quad i, j = 0, 1 \quad (17)$$

Below, this result will be used to calculate the temperature of the radiation of massless scalar and Dirac fields.

A. Thermal radiation of massless scalar fields

In order to identify the temperature of the two-dimensional black hole radiating scalar fields, we compare (15) and (17):

$$\frac{c f'^2(r_+)}{192\pi} = \frac{\Upsilon}{2\pi} \int_0^\infty \omega \langle n_\omega^{\text{BE}}(T) \rangle d\omega, \quad (18)$$

where the temperature T is contained in the Bose-Einstein distribution $\langle n_\omega^{\text{BE}}(T) \rangle$. This last equation shows that presence of the central charge in (15) is essential because it is somehow a measure of the number of degrees of freedom of the system [15]. As it is additive, the central charge of a system of scalar fields is exactly the number of them $c = \Upsilon$, and hence the temperature is

$$T = \frac{1}{4\pi} f'(r_+). \quad (19)$$

B. Thermal radiation of massless Dirac fields

In his original paper, Hawking [1] claims that besides scalar fields, a black hole could also radiate massless fermionic fields. In spite of the fact that fermions obey a different distribution, the same temperature (19) would be found. Several works have explicitly shown this result from different approaches: the uniformly accelerated detector in vacuum method, shown separately by Davies [16] and Unruh [17], was extended to treat the case where the field seen by the accelerated observer is a spin-1/2 Dirac field [18]; the Dirac equation was studied in a black hole background and provides a derivation of the Hawking temperature [19]; and in the three-dimensional black hole the Hawking radiation of Dirac fields agrees with the one obtained from the scalar field case [20]. In our approach, the information about the type of fields used for AdS/CFT calculations is in the central charge. The central charge of one Dirac field is 1/2, hence each field contribute with this quantity to the total central charge; so for Υ Dirac fields we have $c = \Upsilon/2$. Using this and replacing Bose-Einstein by Fermi-Dirac distribution, equation (18) becomes

$$\frac{f'^2(r_+)}{384\pi} = \frac{1}{2\pi} \int_0^\infty \omega \langle n_\omega^{\text{FD}}(T) \rangle d\omega. \quad (20)$$

Even though the left hand side decreases by a half compared with the scalar case (18), the temperature is the same because in (20) we use the Fermi-Dirac distribution and the integral on the right hand decreases by a half as well. Therefore, the temperature from (20) is also given by (19).

V. CONCLUSIONS

AdS/CFT correspondence allows computation of the Hawking temperature for a 2D black hole by direct comparison of both sides of the duality. In the AdS gravity side, the finite boundary energy-momentum tensor was obtained using the Fefferman-Graham expansion. The induced theory on the boundary is an auxiliary Liouville field whose stress tensor depends of the metric and the central charge of the two-dimensional CFT. As this number characterizes the field theory it contains the information about the number and the type of fields radiated by the black hole. The thermal energy-momentum tensor was found, which has the information about the fields radiated in the statistical distribution. By comparing the energy-momentum tensor calculated by both methods, it was possible to identify of the black hole temperature. The method presented in this paper allows for the determination of the temperature of a black hole radiating massless scalar fields and Dirac fields without the necessity of solving the corresponding Klein-Gordon and Dirac equations on a curved spacetime. In both cases the temperature agrees with the one found by Hawking.

The next step is the extension of the procedure shown in this work to a four-dimensional black hole. An important difference will appear for the $\text{AdS}_5/\text{CFT}_4$ calculations because, for a given string theory, the correspondence establishes exactly what the CFT is. For instance, Type IIB string theory on AdS_5 has a corresponding CFT

which is the $\mathcal{N} = 4$ super-Yang-Mills gauge theory. The number of fields is also already determined: there are six scalar fields, two Dirac fields, and one Yang-Mills field; in the large N limit, each of the N^2 parameters of $\text{SU}(N)$ contribute with a degree of freedom; therefore, each field will contribute with a factor N^2 to the radiation energy-momentum tensor. Additionally, the relation between the AdS radius and the central charge used in the two-dimensional case would be replaced by parameters as fundamental as the length of the string; therefore, a non-trivial test of AdS/CFT would be its capability to provide the temperature for a four-dimensional black hole.

Acknowledgments

I would like to thank M. Bañados for his advice and many enlightening conversations. Also thanks to A. Castro, whose work was crucial to understand subtle details of this project. Special thanks to R. Olea, A. Reisenegger, and J.P. Staforelli, for discussions which contributed enormously to this work. Important remarks on an earlier version of the manuscript by M. Ahmad, M. Berger, M.J. Cordero, V.A. Kostelecký, and N. Poplawski, are also gratefully acknowledge. Finally, thanks to Centro de Estudios Científicos in Valdivia, Chile for hospitality during the initial stages of this work.

-
- [1] S.W. Hawking, “*Particle Creation by Black Holes*”, Commun. Math. Phys. **43** (1975) 199.
 - [2] J.M. Maldacena, “*The Large N Limit of Superconformal Field Theories and Supergravity*”, Adv. Theor. Math. Phys. **2** (1998) 231 [hep-th/9711200].
 - [3] S.M. Christensen and S.A. Fulling, “*Trace Anomalies and the Hawking Effect*”, Phys. Rev. D **15** (1977) 2088.
 - [4] A. Castro “*Hawking radiation and AdS/CFT conjecture*”, B.S. Thesis (Pontificia Universidad Católica de Chile, 2004).
 - [5] A. Wipf, “*Quantum Fields near Black Holes*”, [hep-th/9801025].
 - [6] C. Fefferman and S.A. Graham, “*Conformal Invariants*”, in Elie Cartan et les Mathématiques d’aujourd’hui (Astérisque, 1985) 95.
 - [7] S. de Haro, K. Skenderis and S. Solodukhin, “*Holographic Reconstruction of Spacetime and Renormalization in the AdS/CFT Correspondence*”, Commun. Math. Phys. **217** (2004) 595 [hep-th/0002230].
 - [8] K. Skenderis and S. Solodukhin, “*Quantum Effective action from AdS/CFT Correspondence*”, Phys. Lett. B **472** (2000) 316 [hep-th/9910023].
 - [9] J.D. Brown and M. Henneaux, “*Central charges in the canonical realization of asymptotic symmetries: An example from three-dimensional gravity*”, Commun. Math. Phys. **104** (1986) 207.
 - [10] M. Henningson and K. Skenderis, “*The Holographic Weyl Anomaly*”, JHEP **9807** (1998) 023 [hep-th/9806087].
 - [11] M. Henningson and K. Skenderis, “*Holography and the Weyl Anomaly*”, (1998) [hep-th/9812032].
 - [12] P.C.W. Davies, S.A. Fulling and W.G. Unruh, “*Energy-momentum tensor near an evaporating black hole*”, Phys. Rev. D **13** (1976) 2720.
 - [13] A.A. Belavin, A.M. Polyakov and A.B. Zamolodchikov, “*Infinite conformal symmetry in two-dimensional quantum field theory*”, Nucl. Phys. B **241** (1984) 333.
 - [14] J.S. Díaz “*The energy-momentum tensor from AdS/CFT correspondence*”, B.S. Thesis (Pontificia Universidad Católica de Chile, 2005).
 - [15] P. Di Francesco, P. Mathieu and D. Sénéchal, “*Conformal Field Theory*”, Springer (1997)
 - [16] P.C.W. Davies, “*Scalar production in Schwarzschild and Rindler metrics*”, J. Phys. A **8** (1975) 609.
 - [17] W. G. Unruh, “*Notes on black-hole evaporation*”, Phys. Rev. D **14** (1976) 870.
 - [18] P.M. Alsing and P.W. Milonni, “*Simplified derivation of the Hawking-Unruh temperature for an accelerated observer in vacuum*”, Am. J. Phys. **72** (2004) 1524 [quant-ph/0401170].
 - [19] A. Lasenby, C. Doran and S. Gull, “*Gravity, gauge theories and geometric algebra*”, Phil. Trans. Roy. Soc. Lond. A **356** (1998) 487 [gr-qc/0405033].
 - [20] S. Hyun, Y.-S. Song and J.H. Yee, “*Hawking Radiation of Dirac Fields in the (2+1)-Dimensional Black Hole spacetime*”, Phys. Rev. D **51** (1995) 1787 [hep-th/9409047].